Exponents and Radicals

A. Evaluate each of the following: Show all your work

a) $81^{\frac{1}{2}} = (3^4)^{\frac{1}{2}} = 3^{\frac{4}{2}} = 3^2$

b) $\left(\dfrac{81}{196}\right)^{\frac{1}{2}} = \left(\dfrac{3^4}{2^2 7^2}\right)^{\frac{1}{2}} = \dfrac{3^{\frac{4}{2}}}{2^{\frac{2}{2}} 7^{\frac{2}{2}}} = \dfrac{3^2}{2 \cdot 7}$

c) $\left(\dfrac{1000}{27}\right)^{-\frac{2}{3}} = \left(\dfrac{2^3 5^3}{3^3}\right)^{-\frac{2}{3}} = \dfrac{2^{\frac{-6}{3}} 5^{\frac{-6}{3}}}{3^{\frac{-6}{3}}} = \dfrac{2^{-2} 5^{-2}}{3^{-2}} = \dfrac{3^3}{2^2 5^2}$

d) $(-243)^{-\frac{6}{5}} = \left((-3)^5\right)^{\frac{-6}{5}} = (-3)^{\frac{-30}{5}} = (-3)^{-6} = \dfrac{1}{(-3)^6} = \dfrac{1}{3^6}$

e) $\left(\dfrac{1}{128}\right)^{\frac{4}{7}} = \left(\dfrac{1}{2^7}\right)^{\frac{4}{7}} = \dfrac{1}{2^{7 \cdot \frac{4}{7}}} = \dfrac{1}{2^4}$

B. Simplify each of the following:

a) $x^{\frac{3}{7}} x^{\frac{9}{7}} = x^{\frac{12}{7}}$

b) $x^{\frac{2}{3}} x^{\frac{3}{5}} = x^{\frac{10}{15}} x^{\frac{9}{15}} = x^{\frac{19}{15}}$

c) $\left(x^{\frac{3}{5}}\right)^{\frac{4}{7}} = x^{\frac{12}{35}}$

d) $\dfrac{x^{\frac{4}{5}}}{x^{\frac{2}{3}}} = \dfrac{x^{\frac{12}{15}}}{x^{\frac{10}{15}}} = x^{\frac{2}{15}}$

e) $\dfrac{x \cdot x^{\frac{2}{3}} \cdot x^{\frac{5}{6}}}{x^{-4} \cdot x^{\frac{1}{2}}} = \dfrac{x^{\frac{6}{6}} \cdot x^{\frac{4}{6}} \cdot x^{\frac{5}{6}}}{x^{-\frac{24}{6}} \cdot x^{\frac{3}{6}}} = \dfrac{x^{\frac{6}{6}} \cdot x^{\frac{4}{6}} \cdot x^{\frac{5}{6}} \cdot x^{\frac{24}{6}}}{x^{\frac{3}{6}}} = x^{\frac{36}{6}} = x^6$

f) $3x^{-\frac{1}{3}}\left(2x^{\frac{1}{2}} - 3x\right) = 3x^{-\frac{2}{6}}\left(2x^{\frac{3}{6}} - 3x^{\frac{6}{6}}\right) = 6x^{\frac{1}{6}} - 9x^{\frac{4}{6}}$

g) $\dfrac{x^{5a} \cdot x^{7a}}{x^{4a}} = \dfrac{x^{12a}}{x^{4a}} = x^{8a}$

h) $\left(a^{2x-y}\right)\left(a^{2x+y}\right) = a^{4x}$

C. Write each of the following in simpliest form:

a) $\sqrt{162} = \sqrt{2 \cdot 3^4} = 3^2 \sqrt{2}$ b) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$ c) $\sqrt[4]{2^6 x^{12} y^{15}} = 2x^3 y^3 \sqrt[4]{2^2 y^3}$

d) $\sqrt[3]{81 y^4 y^{11}} = \sqrt[3]{3^4 y^{15}} = 3y^5$ e) $\sqrt[3]{\dfrac{x^7}{243 y^6}} = \sqrt[3]{\dfrac{x^7}{3^5 y^6}} = \dfrac{\sqrt[3]{x^7}}{\sqrt[3]{3^5 y^6}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{x^2 \sqrt[3]{3x}}{3^2 y^2}$

D. Simplify each of the following radicals:

a) $3\sqrt{5} - 2\sqrt{5} + 7\sqrt{5} = 8\sqrt{5}$

b) $-4\sqrt{6} + 3\sqrt{2} - 5\sqrt{6} + 9\sqrt{2} = -9\sqrt{6} + 12\sqrt{2}$

c) $\sqrt{27} + 2\sqrt{75} = \sqrt{3^3} + 2\sqrt{3 \cdot 5^2} = 3\sqrt{3} + 10\sqrt{3} = 13\sqrt{3}$

d) $3\sqrt[4]{80} + 6\sqrt[4]{405} = 3\sqrt[4]{2^4 \cdot 5} + 6\sqrt[4]{3^4 \cdot 5} = 3 \cdot 2\sqrt[4]{5} + 6 \cdot 3\sqrt[4]{5} = 6\sqrt[4]{5} + 18\sqrt[4]{5} = 24\sqrt[4]{5}$

e) $\sqrt{12} + 3\sqrt[3]{24} = \sqrt{2^2 \cdot 3} + 3\sqrt[3]{2^3 \cdot 3} = 2\sqrt{3} + 3 \cdot 2\sqrt[3]{3} = 2\sqrt{3} + 6\sqrt[3]{3}$

f) $\sqrt{15} \cdot \sqrt{5} = \sqrt{3 \cdot 5} \cdot \sqrt{5} = \sqrt{3 \cdot 5^2} = 5\sqrt{3}$

g) $\sqrt{3x} \cdot \sqrt[3]{3} = (3x)^{\frac{1}{2}} (3)^{\frac{1}{3}} = 3^{\frac{1}{2}} x^{\frac{1}{2}} 3^{\frac{1}{3}} = 3^{\frac{3}{6}} x^{\frac{3}{6}} 3^{\frac{2}{6}} = 3^{\frac{5}{6}} x^{\frac{3}{6}} = \sqrt[6]{3^5 x^3}$

h) $2\sqrt[5]{8} \cdot -2\sqrt[5]{20} = 2\sqrt[5]{2^3} \cdot -2\sqrt[5]{2^2 \cdot 5} = -4\sqrt[5]{2^5 \cdot 5} = -4 \cdot 2\sqrt[5]{5} = -5\sqrt[5]{5}$

i) $\dfrac{\sqrt{60}}{\sqrt{15}} = \dfrac{\sqrt{2^2 \cdot 3 \cdot 5}}{\sqrt{3 \cdot 5}} = \sqrt{2^2} = 2$

j) $\dfrac{\sqrt[3]{6}}{\sqrt[3]{15}} = \dfrac{\sqrt[3]{2 \cdot 3}}{\sqrt[3]{3 \cdot 5}} = \dfrac{\sqrt[3]{2}}{\sqrt[3]{5}} \cdot \dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} = \dfrac{\sqrt[3]{2 \cdot 5^2}}{5}$

k) $\left(3 - \sqrt{5}\right)\left(3 + \sqrt{5}\right) = 9 + 3\sqrt{5} - 3\sqrt{5} - 5 = 4$

l) $\left(\sqrt[3]{7} - 3\right)\left(\sqrt[3]{7} + 2\right) = \sqrt[3]{7^2} + 2\sqrt[3]{7} - 3\sqrt[3]{7} - 6 = \sqrt[3]{7^2} - \sqrt[3]{7} - 6$

m) $\dfrac{3\sqrt{2}}{2\sqrt{5}} = \dfrac{3\sqrt{2}}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{3\sqrt{2 \cdot 5}}{2 \cdot 5}$

n) $\dfrac{2}{3 + \sqrt{5}} = \dfrac{2}{\left(3 + \sqrt{5}\right)} \cdot \dfrac{\left(3 - \sqrt{5}\right)}{\left(3 - \sqrt{5}\right)} = \dfrac{6 - 2\sqrt{5}}{9 - 5} = \dfrac{6 - 2\sqrt{5}}{4} = \dfrac{3 - \sqrt{5}}{2}$